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Richland, WA 99352

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THE DRY WELL FREQUENCY RESPONSE EQUATION: A CRITICAL REVIEW OF ITS BASIS AND DERIVATION

H. A. Forrester
K. A. Gasper
R. E. Isaacson
A. H. Lu
S. J. Phillips
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RHO-RE-EV-11

THE DRY WELL FREQUENCY RESPONSE EQUATION: A CRITICAL REVIEW OF ITS BASIS AND DERIVATION

Review Committee Members

H. A. Forrester
K. A. Gasper
R. E. Isaacson
A. H. Lu
S. J. Phillips
R. C. Routson
W. W. Schulz (Chairman)

Research and Engineering
Health, Safety and Environment

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ABSTRACT

Dry wells are drilled around each of the 149 single-shell underground waste tanks at the Hanford Site. The recently derived "Dry Well Radioactivity Response Equation" (RHO-ST-34) is a scientifically valid and technically correct basis for establishing the frequency of periodic monitoring of radiation levels in these dry wells. This latter conclusion is the principal finding of a special Rockwell Hanford Operations Review Committee commissioned to perform an independent, critical, and comprehensive technical analysis of the scientific and mathematical logic and methodology underlying the dry well radioactivity response equation. As part of the deliberations of the Review Committee, an analytical model (diffusion model) of the motion of the plume from a leaking single-shell tank was derived by rigorous solution of the partial differential equations governing the motion of fluids through soils. Of great significance, the diffusion model yields results essentially equivalent to those produced by the dry well radioactivity response equation.

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CONTENTS

TABLES:

INTRODUCTION

Solidified high-level wastes containing varying amounts of liquid are stored in 149 underground single-shell tanks at the Hanford Site. These tanks are monitored routinely to detect promptly any liquid leaks. Monitoring systems embrace equipment to measure both liquid levels in the tanks and gamma activity in steel-encased dry wells drilled around each tank.

Coincident with the solidification of the major portion of the Hanford defense high-level liquid waste, the dry well monitoring system has become more important as a primary leak detection device. A critical part of the overall dry well monitoring scheme is the establishment of a suitable tank-by-tank monitoring frequency. This fact was recognized in the so-called Catlin report (Catlin, 1980), which stated: "Formal criteria are needed to redetermine the surveillance frequency for each tank and the development of such criteria is recommended, taking into account pertinent technical factors such as available monitoring systems, tank contents, and their relative mobility." The Catlin report motivated a comprehensive study within Rockwell Hanford Operations (Rockwell) that culminated recently with the issuance of the report entitled "A Scientific Basis for Establishing Dry Well-Monitoring Frequencies" (Isaacson and Gasper, 1981).

Rockwell has recently changed over to the new dry well monitoring frequency schedule recommended in the Isaacson-Gasper report. Anticipating this action and recognizing its importance and significance, Rockwell early-on* commissioned a special in-house Review Committee to perform an independent critical and comprehensive analysis of the methodology, logic, and assumptions used by Isaacson to derive a dry well radioactivity response equation (dry well response equation). The Rockwell Review Committee (Review Committee) was empowered to seek (and did seek) opinions and counsel of outside experts and was also chartered to investigate alternative approaches to deriving a dry well response equation. The Review Committee has now completed its deliberations and its findings are summarized in this report.†

The work of this Review Committee was funded by the Surveillance and Maintenance (WA) end function of the Waste Management Program.

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*While the Isaacson-Gasper report was still in its first draft form.

†Mr. Isaacson joined the Review Committee for the last half of its tenure. Membership on the Review Committee enabled Mr. Isaacson to be fully informed about the principal findings and recommendations of the committee and to incorporate them in the final version of the Isaacson-Gasper document.

SUMMARY AND CONCLUSIONS

This report presents significant findings and other relevant aspects of a special Review Committee. This Review Committee was convened to perform an independent, critical, and comprehensive technical analysis of the scientific and mathematical logic and methodology underlying the dry well response equation. Motivation for performance of such a review was provided by a recent Rockwell management decision to use the dry well response equation as the basis for setting the frequency of periodic monitoring of radiation levels in dry wells drilled around single-shell underground waste tanks at the Hanford Site.

The Review Committee, composed of seven highly experienced and qualified Rockwell scientists and engineers, was chartered to:

- Perform a detailed evaluation of the validity of the assumptions upon which the dry well response equation is based
- Arrange for critical review of the dry well response equation by qualified external experts and evaluate their findings
- Consider, develop, and evaluate alternative approaches to derivation of a dry well response equation.

On the basis of its extensive deliberations, the Review Committee finds that the dry well response equation is a scientifically valid and technically correct basis for establishing the frequency of periodic monitoring of radiation levels in dry wells. The strongest support for this conclusion is provided by the finding that a rigorously derived analytical model (diffusion model) of the motion of a plume from a leaking tank yields results essentially equivalent to those produced by the dry well response equation. The Review Committee, as well as three external reviewers, concurs that assumptions and approximations used to derive the dry well response equation are generally reasonable and well founded.

REVIEW COMMITTEE: MEMBERS AND ORGANIZATIONAL DETAILS

REVIEW COMMITTEE MEMBERS

Seven senior scientists and engineers (Table 1) from the Research and Engineering and Health, Safety and Environment functions served on the Review Committee. Each member was carefully selected on the basis of his extensive professional experience and qualifications and to ensure

TABLE 1. Makeup of Review Committee.

Name	Title[a]	Rockwell organizational component[a]	Professional discipline
H. A. Forrester	Senior Statistician	Statistics & Mathematics Unit Research & Engineering	Mathematician
K. A. Gasper	Program Engineer	Program Engineering Department Research & Engineering	Nuclear Engineer
R. E. Isaacson[b]	Chief Scientist	Program Engineering Department Research & Engineering	Chemical Engineer
A. H. Lu	Staff Hydrologist	Hydrological Sciences Unit Health, Safety & Environment	Hydrologist
S. J. Phillips	Staff Scientist	Technology Development Unit Research & Engineering	Soil Physicist
R. C. Routson	Staff Soil Scientist	Geological Sciences Unit Health, Safety & Environment	Soil Scientist
W. W. Schulz (chairman)	Chief Scientist	Program Engineering Department Research & Engineering	Chemist

[a]At the time of Review Committee's tenure.

[b]Joined Review Committee midway through its deliberations.

a properly balanced array of scientific and engineering disciplines. Mr. Isaacson, who derived the dry well response equation, joined the Review Committee after it had begun its deliberations.

REVIEW COMMITTEE CHARTER

After considerable discussion, the Review Committee agreed to the charter given below. The Review Committee concluded that risk assessment was not part of its charter and did not attempt to relate dry well monitoring to leak location or to the probability of detecting a leak of a given size.

The Review Committee's charter called for an independent, critical, and comprehensive technical analysis and review of the scientific and mathematical logic and methodology underlying the dry well response equation. This review is to:

- List assumptions upon which the dry well response equation is based
- Perform a detailed in-house (Rockwell) evaluation of the validity of these assumptions
- Arrange for critical review of dry well response equation (bases, logic, etc.) by qualified external experts; evaluate findings of external reviewers
- Consider, develop, and evaluate alternative approaches to derivation of a dry well response equation
- Prepare a report summarizing deliberations and findings of Review Committee.

PROVISION FOR EXTERNAL (NON-ROCKWELL) REVIEW

From the outset, the Review Committee recognized clearly the need for an objective appraisal of the rationale and methodology employed by Isaacson in deriving the dry well response equation by competent and qualified scientists outside Rockwell. Accordingly, arrangements were made and completed to have three experienced professionals* of Battelle's

*Consent of these reviewers to list their names in this report of the Review Committee was not sought or obtained prior to preparation of their comments. For this reason, the external reviewers are identified only by number in this report.

Pacific Northwest Laboratory review in detail a draft of the Isaacson-Gasper report before its publication. The individuals chosen by the Review Committee were selected on the basis of their known expert in-depth knowledge of one or more aspects of the equipment and methods used to detect radiation in Hanford dry wells and of the hydrological and other characteristics of Hanford soils and sediments.

Edited versions of the comments of the three external reviewers are presented in Appendix A;* summaries of the Review Committee's response to the issues raised by the external reviewers are also reported in Appendix A. As noted in the Review Committee's chronology, the comments of the external reviewers and the response of the Review Committee were available prior to the formal publication of RHO-ST-34 (Isaacson and Gasper, 1981).

CHRONOLOGY OF INTERNAL (ROCKWELL) REVIEW

Significant events during the life of the Review Committee are recorded below:

February 1981	Review Committee formally commissioned, selected, and organized.
April 1981	Charter of Review Committee formulated and agreed to.
February-August 1981	Review Committee meetings on an approximate biweekly schedule.
June 1981	Alternative (analytical) model and approach to "dry well response equation" developed and documented.
June 1981	Received comments from External Reviewers 1 and 2.
July 1981	Received comments from External Reviewer 3.
July 1981	Review Committee conducted intensive review of comments from External Reviewers 1 and 2.
August 1981	Review Committee conducted intensive review of comments from External Reviewer 3.

*Only some personal comments not germane to the technical material have been deleted from the versions.

December 1981	RHO-ST-34, A Scientific Basis for Establishing Dry Well-Monitoring Frequencies" issued.
April 1982	Rough draft of final report of Review Committee completed.
October 1982	Final report of Review Committee issued.

DRY WELL RESPONSE EQUATION: BASES AND ASSUMPTIONS

Dry wells have been drilled around each of the Hanford single-shell waste tanks (Figure 1). Sensitive radiation detection equipment is periodically inserted in these wells to detect any radioactivity that may have leaked from the tanks. The dry well reponse equation was derived to define a scientific basis for establishing the frequency of monitoring dry wells for the approach of radioactivity from a leaking tank. Monitoring frequency depends on the response characteristics of the radiation detection system.

Isaacson and Gasper (1982) state that the recommended principal criteria for establishing the frequency of monitoring dry wells are:

"1. The monitoring interval shall be established to assure that the incremental volume of waste that is released, between the time of first detecting a significant increase in count rate (20 c/s [counts per second, or counts/sec] above baseline) and the time that the immediately preceding radiation survey was made of that dry well, will not exceed 1,375 gal (i.e., equivalent to a liquid-level decrease in the waste tank of 0.5 in.).

2. When the count in a dry well exceeds the baseline level by 20 counts/sec, the monitoring frequency will be increased in that dry well and in adjacent dry wells.

3. The minimum monitoring interval for a dry well shall ensure that the count rate does not exceed an action level of 160 counts/sec above the baseline level within the period between successive readings for more than 10% of the possible leaks within the nearest range of that dry well."

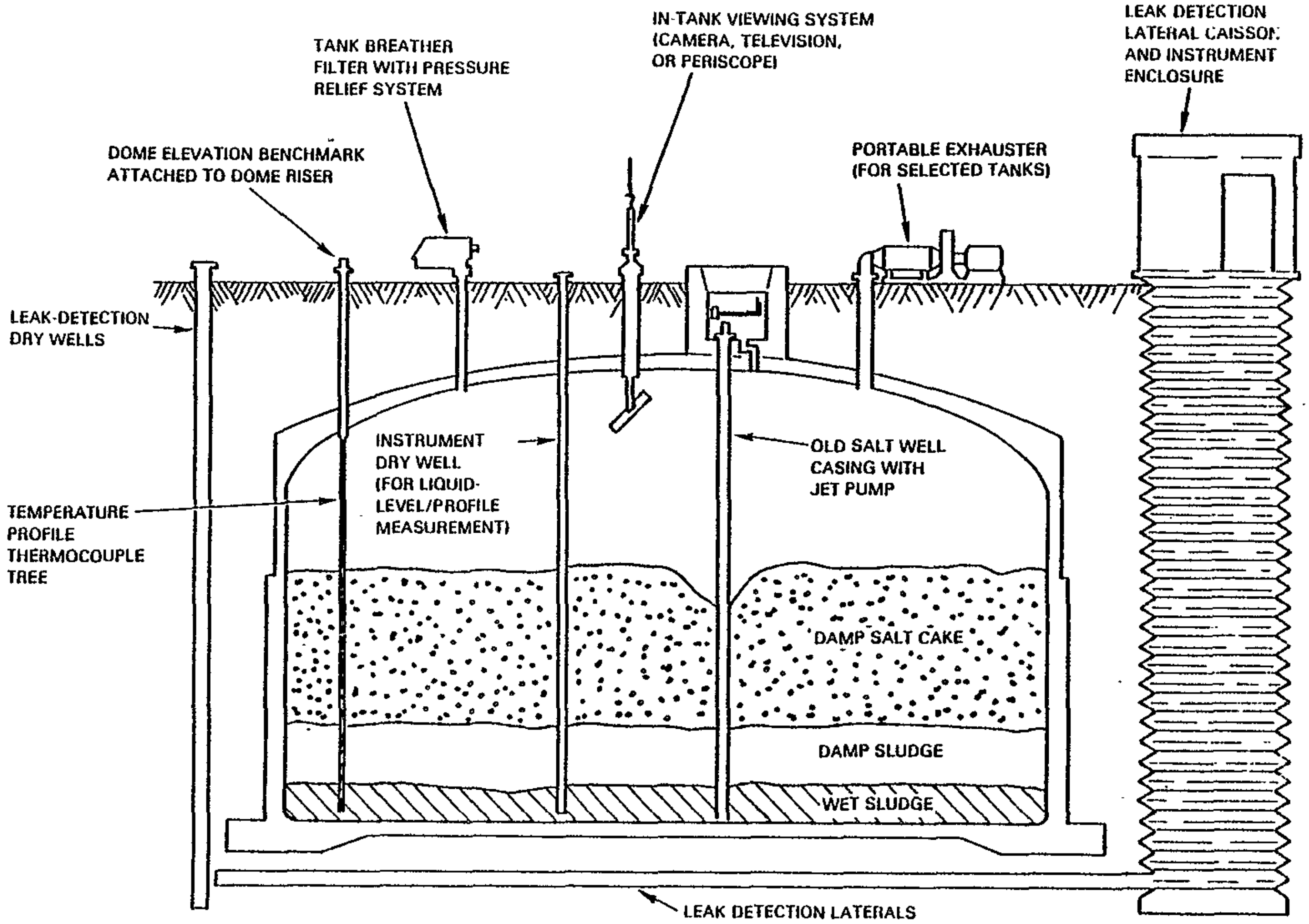


FIGURE 1. High-Level Waste Tank Surveillance Instrumentation. (Leak detection laterals exist only under certain tanks.)

Equation 1 is the dry well response equation derived by Isaacson:

$$\Delta\tau_{\Delta d} = \left(b^{3} - \left\{ b - \left[\frac{\lambda' \bar{m}}{c} \right] \left[\log_{10} \frac{n_t}{n_{t-\Delta t}} \right) + \left(\frac{\Delta t}{368} \log_{10} 2 \right) \right] \right\}^{3} \right) \frac{g'g}{s} \quad (1)$$

Most of the terms in Equation 1 are defined in Figures 2 and 3, which illustrate the fundamental aspects of the Isaacson model. Of the remaining terms:

- n_t = count rate at time t (t = time when the count rate is at the action level of 160 counts/sec above background level).
- $t-\Delta t$ = time when count rate is at the alert level of 20 counts/sec above background level.
- $n_{t-\Delta t}$ = count rate at time t - Δt.
- c = detector calibration factor (0.918) for standard scintillation probe that relates dose rate in R/hr to count rate in c/sec.
- $l_{\bar{m}}$ = effective mean tenth-value thickness of soil. (Note: The thickness of soil that reduces dose rate by one-tenth equals the effective tenth-value thickness.)

The dry well response equation is derived from the variation in dose rate (R/hr) as a function of source strength, variations in dose attenuation by the soil as the radioactive waste front approaches the dry well, response of the radiation detector (counts/sec) as the dose rate changes (instrument calibration), distance of the dry well from the tank leak source, leak rate, geometry of the soil wetted by the leaking waste, and the hydrologic properties of the soil.

The six key assumptions made by Isaacson in deriving the dry well response equation are listed in Table 2. A major task of the Review Committee was, as previously stated, to determine if these assumptions are valid and sufficiently conservative.

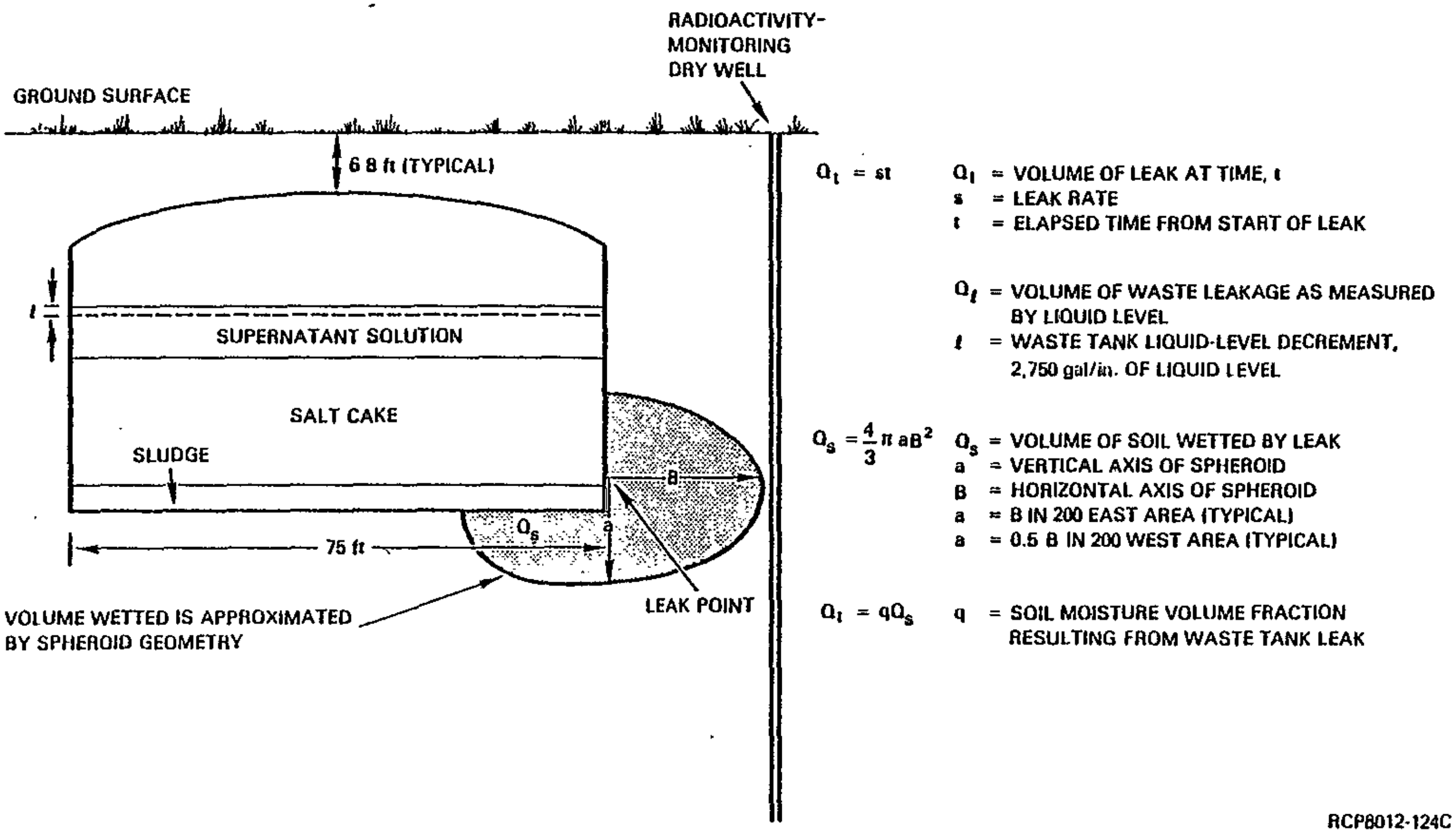


FIGURE 2. Volumetric and Geometric Relationships of Waste Tank Leaks.

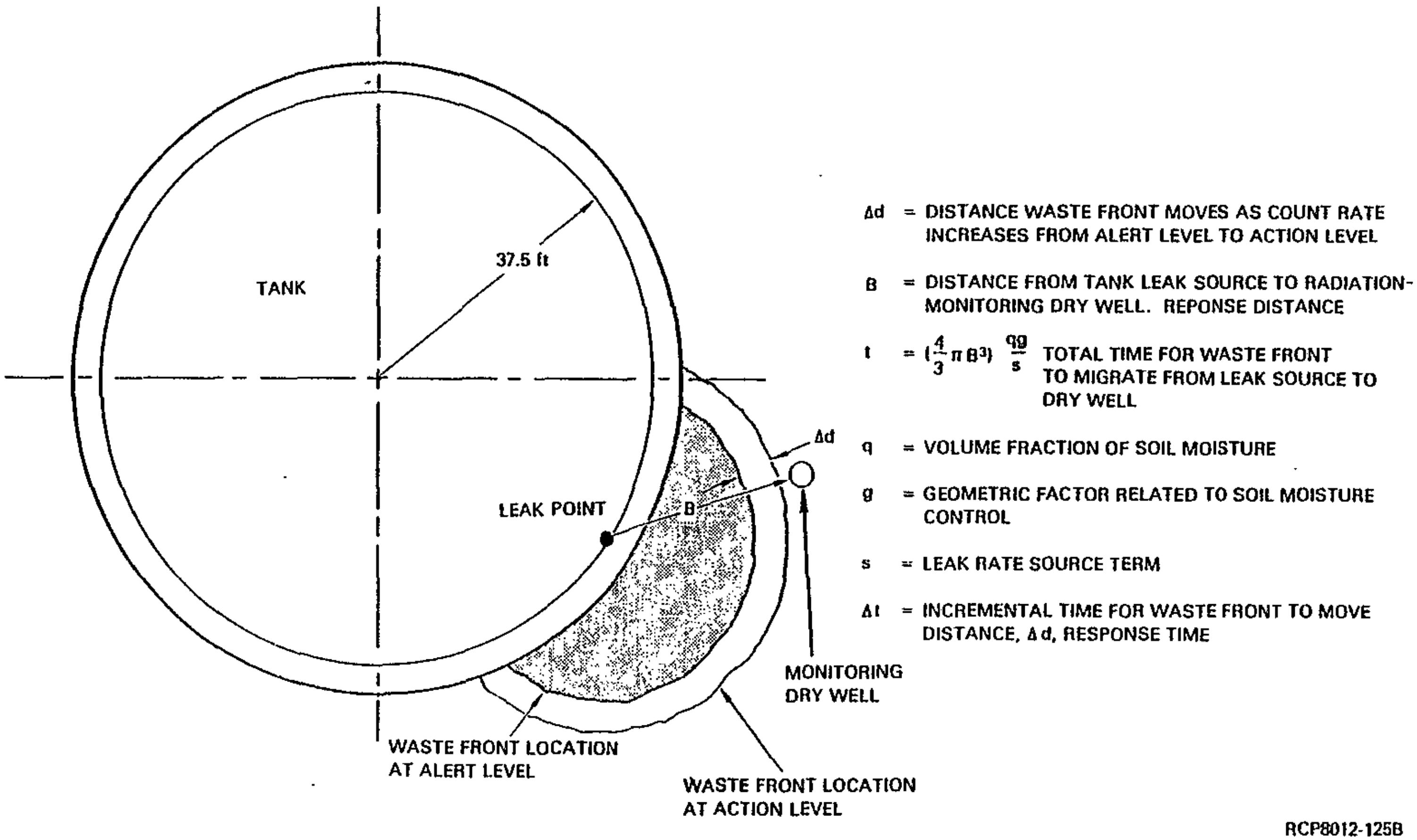


FIGURE 3. Factors Affecting Response Time of Radiation Monitoring Dry Wells to High-Level Waste Tank Leaks.

TABLE 2. Key Assumptions Made in Deriving Dry Well Response Equations.

No.	Assumption
1	Leak plumes have approximate spherical geometry.
2	The soil moisture content remains nearly constant during the leak.
3	The volumetric moisture content, q, of the soil is 8%.
4	The maximum tank leak rate, s, is 0.03 gal/min.
5	Radioactive ruthenium is a satisfactory waste front tracer.
6	The leak occurs on the perimeter of a tank at a point ~12 ft on either side of the nearest distance to a dry well.

REVIEW OF ASSUMPTIONS

ASSUMPTION 1. LEAK PLUMES HAVE APPROXIMATE SPHERICAL GEOMETRY

In their report, Isaacson and Gasper state, "The geometry of an oblate spheroid was chosen for 200 West Area after examining the 241-T-106 plume. The cubic root dependence of the radius to total volume leaked may not apply in sediments that exhibit stronger layering. In strongly layered soils, the contamination front will move much further and faster. For this, one might hypothesize a square root dependence of count rate versus time."

Need/Basis for Assumption

After joining the Review Committee, Isaacson provided some additional information regarding the basis for the assumption that the geometry of leak plumes in the 200 East Area can be approximated by a sphere while the geometry of such plumes in 200 West Area can be approximated by an oblate spheroid whose diameter is twice its height. Isaacson stated that these assumptions were based not only on the characterization of actual leak plumes, particularly that of the 241-T-106 Tank leak in 200 West Area,* but also on results of infiltration studies such as those depicted in Figure 3 (a and b) of RHO-ST-14 (Routson et al., 1979). Copies of these figures are reproduced in this report as Figure 4(a) and (b).

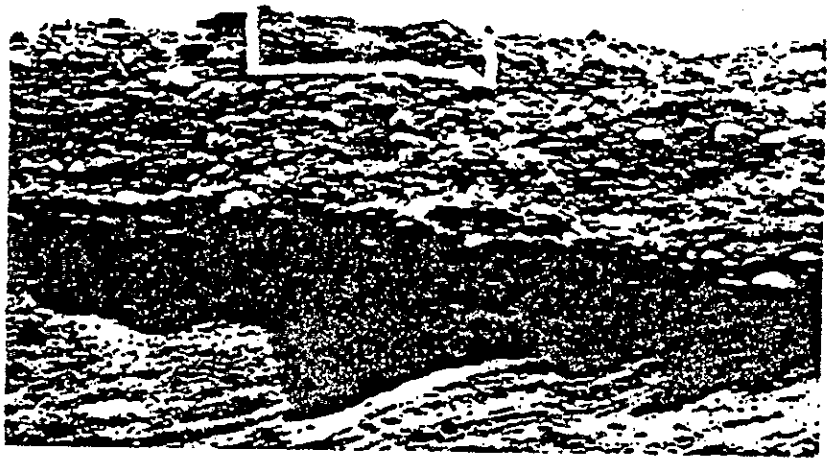
Concerning the plume shown in Figure 4(b), Isaacson points out that its shape illustrates the generality of assumptions about the geometry of plumes from actual wastes in a fluvial deposit of graded sediments. Where fine-textured sediments overlie coarser materials, liquids will be moved preferentially in a horizontal direction by capillary forces. Note, for example, the "tail" on the wetted plume in Figure 4(b) where a fine-grained sediment is between two coarse layers. When leak rates are small and there is insufficient liquid to saturate the sediments, the available moisture will be retained by those layers having the highest capillary potential. In such cases, the leak plume will have the dimensions of the controlling layer and will depend on the total volume of liquid available.

Isaacson emphasized that when leak rates are very low, the capillary potential of the fine-grained sediments provides a high degree of horizontal control of the leak plume geometry. Isaacson noted that arguments about the spherical nature of leak plumes at Hanford are supported by material in the text by Hillel (1971).

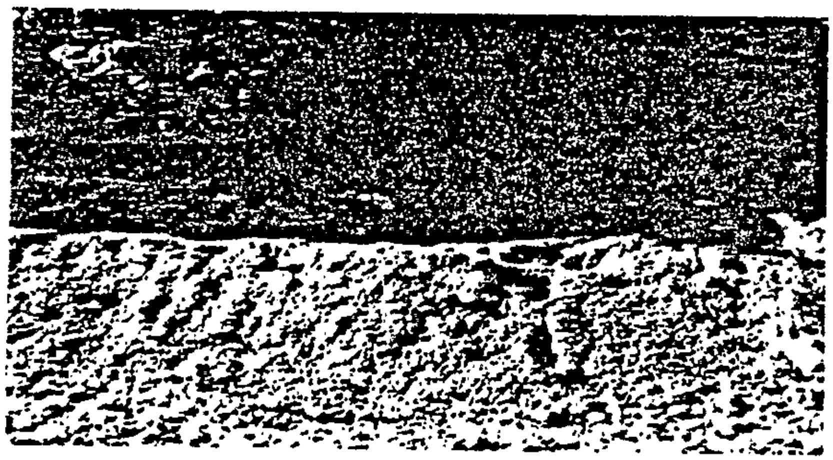
Beyond Isaacson's input, the Review Committee noted that, in general, the morphology of the wetting front as liquid infiltrates into glaciofluvial sediments is a function of numerous complex and interrelated variables. These include: (1) location and shape of the leak in the tank wall; (2) liquid properties (e.g., viscosity, surface tension, hydraulic head, etc.); (3) fluid transport within the tank (e.g., quantities and location of liquid and solid waste within the tank, horizontal and vertical components of fluid transported in the tank); and (4) soil properties (conductivity, diffusivity, hydraulic gradients, initial and incremental liquid contents, retentivity, stratigraphy, isotropic nature, and homogeneity). The stratigraphy of undisturbed geologic media extending below and laterally from the high-level waste storage tanks at the Hanford Site is known to vary from location to location (Price and Fecht, 1976).

The Review Committee agreed that any program to characterize in full the stratigraphy of the sediments under each waste storage tank at the Hanford Site would be both extensive and expensive. Further, even if the soil stratigraphy were fully characterized, evaluation of wetting

*Lu (1980) recently reported results of computer-assisted modeling of the transport of liquid and radionuclides from the 241-T-106 Tank leak through vadose zone soils. His results indicate that the leak plume has the shape of an oblate spheroid with an anisotropic ratio of 15:1.



(a) After 6 hours



(b) After 24 hours

FIGURE 4. Typical Horizontal and Vertical Movement of of Liquids in Hanford Formation Sediments Under Partially Saturated Conditions.

front morphology under a variety of potential tank leak conditions would be even more difficult. Thus, some simplifying assumption (e.g., approximate spherical shape) about leak plume geometry is warranted.

Validity of Assumption 1

In assessing the validity of assumption 1, the Review Committee noted that, because of the anisotropic and heterogeneous characteristics of the geologic media and because of the forces (matric and gravity) acting on and within the flow domain, the morphology of the wetting front of a liquid leaked from a waste tank can change with time over the duration of a leak. Thus, during initial point source infiltration into unsaturated isotropic homogeneous geologic media, spheroidal morphology may indeed prevail. After a longer time, however, a nearly PROLATE spheroid with the vertical axis longer than the horizontal can result (Hillel, 1980). In media of layered sequence of texturally different materials, such as is common to the Hanford Site, an extremely complex morphology that varies over time can result (Miller and Gardner, 1962). Also, preferential flow channels that may transmit liquid at a much higher rate than under isotropic homogeneous infiltration conditions have been observed during vertical infiltration in layered geological media (Raats, 1973). Such flow channels, if they occur in Hanford sediments, cannot, of course, be adequately represented by a plume with spherical shape.

The Review Committee also discussed advantages and disadvantages of assuming only one kind of wetting front morphology (i.e., either perfect sphere or oblate spheroid) in deriving the dry well response equation. An important consideration here is that the gamma energy flux from radionuclides in a leak zone to a detector located at any point in a dry well is more attenuated by the soil in a zone of spherical shape than one in the form of an oblate spheroid. Although not examined in detail, it appears that a response equation derived for plumes of assumed oblate spheroid (with diameter twice the height) may predict that dry wells should be monitored at a frequency slightly greater than that needed to meet the three principal criteria laid down by Isaacson and Gasper (1981). Conversely, a response equation derived for plumes of perfect spherical shape may predict a dry well monitoring schedule slightly less frequent than required to comply with the principal leak detection criteria.

Review Committee Findings

From their deliberations, the majority of the Review Committee found that the assumption of spherical and oblate spheroidal geometry of plumes of leaks from tanks in the 200 East and 200 West Areas, respectively, is reasonable and acceptable. This finding is based upon:

- A recognized need for a simplifying assumption about leak plume shape

- The shape of the plume from the 241-T-106 Tank leak
- Experimental data reported by Routson et al. (1979), and also in TID-2-6431 (1973)
- Lack of any overwhelming reason to choose a geometry other than approximate spherical.

ASSUMPTION 2. THE SOIL MOISTURE CONTENT REMAINS CONSTANT DURING THE LEAK

Isaacson and Gasper said, "The soil moisture content is redistributed rapidly when the total moisture content reaches about 14 vol%; an incremental increase of about 8 vol% over average ambient soil conditions. Leakage from the tank is assumed to control leak rate, not soil permeability. Because of the very high capillary potential or soil suction, soil moisture does not accumulate nor is allowed to build up near the tank. If this happened, the time for the water front to reach the dry well would be increased."

Need/Basis for Assumption

Reasonable approximations to the shape of plumes of any liquid leaked from waste storage tanks are provided by assumption 1. There is also a need to establish what forces drive and control transport of leaked liquid through the soil. Assumption 2 is made to satisfy this need.

Again, after joining the Review Committee, Isaacson provided much valuable insight as to the rationale and basis for assumption 2. Isaacson's extended comments, which are quoted here, were made primarily in response to early questions of the Review Committee about the validity of assumption 2. Isaacson's comments follow:*

> "These assumptions pertain to the empirical model used to represent the net effect of soil moisture transport in the calculations and do not describe the actual conditions of moisture transport in the sediments [Note: Emphasis supplied by compiler (WWS) of this report.] However, the results of the calculations are in general agreement with field measurements and are also in agreement with the approach of Green and Ampt as discussed in the text by Hillel (1980).

*Minor editorial changes have been made to the comments submitted by Isaacson to W. W. Schulz to fit them to the format of this report.

"As noted by Hillel, a Darcy-type equation can be applied directly for horizontal infiltration:

$$i = \frac{dI}{dt} = \frac{K(H_o - H_f)}{L_f} \tag{2}$$

where

i = flux into soil and through the transmission zone

I = cumulative infiltration

K = hydraulic conductivity of the transmission zone

H_o = pressure head at the entry surface

H_f = effective pressure head at the wetting front

L_f = distance from the source to the wetting front (i.e., length of wetted zone).

"Green and Ampt assume a uniformly wetted zone extends all the way to the wetted front. [Note: Isaacson's emphasis.] Under these conditions, the cumulative infiltration I should be equal to the product of the wetting front distance L_f and the wetness increment $\Delta\theta = \theta_t - \theta_i$ where θ_t is the transmission zone wetness during infiltration and θ_i is the initial profile wetness which prevails beyond the wetting front.

"By inspection, i in Equation 2 can be compared to the source term s (tank leak rate) in the dry well response equation [Equation 1]; $\Delta\theta$ can be compared to q, the soil moisture content [Figure 2]; and I can be compared to Q, the total quantity of liquid infiltrated to the soil [Figure 2]. Thus, $I = s\ idt = it = st$ and, therefore, $I = Q = st$.

"While arguments may be made for alternative models, this model is effective because the soil suction (capillary potential or matric potential) is so large that the moisture is distributed in the moisture-deficient soils as rapidly as it is being supplied by the limiting tank leak. Physically it is difficult under these conditions to build up an accumulation near the tank.

"A practical refinement might be in order to better relate the 'dry well response' equation to the 'real world.' This can be done by redefining the term q to relate to the soil moisture conditions prior to the leak and the apparent increment of moisture added as a consequence of the leak. Thus, q could be said to be equivalent to $\Delta\Theta$ of Green and Ampt, which is the difference between the transmission zone wetness during infiltration and Θ_i is the initial profile wetness.

"Observations in the soil moisture measurement lysimeters have shown that the residual moisture in the Hanford sediments is about 6 vol%. This is in essential agreement with measurements in tank farm sediments that have not been subjected to artificial moisture. Further, observations have shown that active frontal advance of moisture occurs at a soil moisture content of about 14 vol%. Thus, the difference (14-6) agrees with the arbitrary value of q (8%) that has been used in the generalized equations.

"It should be noted that if the soil moisture content is high (approaching saturation), then the transmissivity of the soil will control the horizontal spread of the liquids. In such cases, the horizontal movement of radioactivity will be impeded in fine grained sediments. Examination of the available soil moisture data in the tank farms shows that the average soil moisture content is about 8 vol%. Thus, movement of water under the force of capillary potential is the dominant mechanism of soil moisture and radionuclide transport.

"While some of the reviewers are concerned that the gradient is not expressed in the dry well response equation, the term q is the equivalent. While conditions of leakage are not in reality steady state, they are very close to steady state when the leak source supplies liquids at a rate that is less than the soil system will disperse under the influence of a large matric potential. Thus, q could be said to represent a steady state soil moisture gradient at the wetting front.

"Whether q is an incremental soil moisture volume or a displaced soil moisture volume, it has been defined as that soil moisture distributed in soil volume Q_s that originated from the leak. Thus, qQ_s is the volume that leaked from the tank and q is the wetness increment $\Delta\Theta$ (Green and Ampt)."

Validity of Assumption

From the Review Committee's inception, the majority of its members could not (and still cannot) accept the statement, "the soil moisture content remains constant during the leak." If this statement were literally true, no gradient would exist and liquid leaked from a tank would not move in the soil toward a dry well. The Review Committee appreciates that Isaacson identified the term q in the dry well response equation as "...a steady state soil moisture gradient at the wetting front."

Several members of the Review Committee pointed out that mathematical formulae exist (Green and Ampt, 1911; Hillel and Gardner, 1970) for computing the rate of advance of a wetting front in soil. These formulae permit calculation of the volume of soil wetted by liquid waste leaking from a tank.

Initially, several Review Committee members felt strongly that in deriving the dry well response equation, Isaacson should have used the Green-Ampt equation rather than making an a priori assumption about the constancy of soil moisture. Further discussion brought out that to use the Green-Ampt equations directly, a suitable value for negative pressure head (matric potential) at the location of the advancing wetting front must be found experimentally or by extrapolation or interpolation from published information. Further, initial ambient moisture content varies in anisotropic, heterogeneous, or layered media such as are characteristic of the nondisturbed geologic media below waste storage tanks at the Hanford Site. Thus, determination of a reliable matric potential to use in the Green-Ampt equations is difficult if not impossible. On this basis and in view of Isaacson's subsequent comments about assumption 2, the Review Committee agrees that assumption 2 is at least reasonable.

The Review Committee also notes that assumption 2 implies that liquid waste moves as a sharp front as it travels toward a dry well. Simple calculations were made to estimate the extent to which dispersion processes could smear out the moving liquid front as it approaches a dry well. Based upon concepts of statistical analysis (Bear, 1972), the standard deviation of Gaussian distribution is:

$$R = (2D\Delta t)^{1/2} \tag{3}$$

and

$$D = \alpha V \tag{4}$$

where

D = dispersion coefficient (typically 10^{-7} cm^2/sec for three dimensional dispersion in unsaturated soils).

V = infiltration velocity, cm/sec

Δt = infiltration time, sec

R = width of liquid waste front, cm

α = dispersivity

were used in these calculations. Estimated values for D and V as a function of the distance of the liquid front from the leak source are shown in Table 3.

TABLE 3. Dispersive Profile of Liquid Plume.

Distance from tank (m)	V (cm/sec)	D (cm^2/sec)
3	1×10^{-4}	1×10^{-4}
5	8×10^{-6}	8×10^{-6}
7	4×10^{-6}	4×10^{-6}
9	8×10^{-7}	8×10^{-7}
11	4×10^{-7}	4×10^{-7}
13	1×10^{-7}	1×10^{-7}

Using these data, the width of a liquid "front" at a distance of about 11 m from the tank would be only about 0.7 cm after 1 wk and 5 cm after 1 yr. These latter values confirm that liquids moving in layered unsaturated Hanford Site sediments do move with a sharp front.

Review Committee Findings

Because of its fundamental importance in the derivation and application of the dry well response equation, the Review Committee spent much time wrestling with assumption 2, its meaning and validity. No clear-cut resolution of the issues involved was obtained. The majority of the Review Committee do not find any overwhelming reason to reject assumption 2 and, therefore, believe primarily by default that it is an acceptable and reasonable plank upon which to base derivation of the dry well response equation.

ASSUMPTION 3. THE VOLUMETRIC MOISTURE CONTENT OF THE SOIL, q, IS 8%

Isaacson and Gasper stated:

"Measured values of soil moisture ranged from 4 vol% to over 20 vol%. Soil moisture moves very, very slowly at total moisture contents below 10 vol% (Ref. 14).* Thus, the assumed value of 8 vol% may result in overestimating the response time; in more cases, it would underestimate it. Underestimating the moisture content would lead to monitoring of those dry wells more frequently than necessary when moisture content of the soil falls below 8 vol%."

*Referenced in this report as Isaacson et al. (1974).

Need/Basis for Assumption

Assumptions 2 and 3 are very closely connected. Thus, assumption 2 is needed as the fundamental basis for a soil moisture displacement model whereby existing soil moisture is displaced by that from a tank leak. Assumption 2 states that soil moisture content is redistributed rapidly when the total moisture content reaches about 14 vol%. Assumption 3 says that such redistribution occurs when there is an incremental moisture content of about 8% over average ambient soil conditions. Stated otherwise, assumption 2 is equivalent to stating that the average ambient soil moisture content is about 6%.

Table 4 lists actual experimentally determined moisture contents of soils in Hanford tank farms. These data represent rounded (to ±0.05%) moisture contents of the upper two sediment types in the tank farms; only these two sediments are believed to be involved in frontal movement of liquid to dry wells. As noted in Table 4, the grand overall average soil moisture content is 9.0%, only 1/3 higher than the value in assumption 3.

TABLE 4. Average Moisture Content of Soils in Hanford Tank Farms.

Tank farm designation (241-)	Moisture content (vol%)[a,b]		
	Depth 1	Depth 2	Average
A	11.0	5.0	8.0
AX	12.5	7.0	9.5
B	10.0	7.0	8.5
BX	9.0	8.0	8.5
BY	10.0	1.0	8.0
C	9.5	5.0	7.0
S	12.5	11.0	12.0
SX	12.5	6.5	8.5
T	12.0	8.5	10.0
TX	9.0	7.0	8.0
TY	10.0	6.5	8.0
U	12.0	8.5	10.0
	Grand average		9.0

[a]Each value in the first two columns is the average of readings from each of the five wells in each tank farm.

[b]Depths represent the centers of two major sediment types as determined from granulometric data.

Knowledgeable members of the Review Committee pointed out that all tank farms have had large volumes of moisture added to them during construction and operation so that their present moisture contents (see Table 4) are considerably higher than their pristine values. Routson and Fecht (1979) reported that undisturbed sediments at the Hanford Site contain only about 1% to 3% moisture by weight.

As with the other assumptions, Isaacson provided valuable comments to the Review Committee on the basis for assumption 3. Isaacson stated:*

> "As discussed under assumption 2, q is the volume fraction of soil that has become filled with liquid as a consequence of a tank leak. It is assumed to be constant over the volume of the leak plume and this is approximately correct because of the controlling effect of the high matric potential of the moisture deficient soils. It must be understood that sufficient liquids must be added to cause frontal movement. The reason that the movement of moisture is slow at the 6% moisture content is that the process is one of redistribution. Since a continuous film of water does not exist, and transport of moisture is primarily in the vapor phase below a moisture content of 6%, the rate of redistribution will be exceedingly slow and is energy dependent. As water is added to a system which has become desiccated, there are two effects which must be considered. One is the surface tension of the liquid and the other is the capillary effect which is related to the size and angles of intergranular spaces as well as the surface tension of the water. In one case, there is a resistance to wetting of the sedimentary particles and in the other there is a capillary potential that holds the liquid until the 'meniscus effect' is satisfied. Capillary potential (suction) may range from 100 to >10,000 cm of water. These effects are evidenced by a strong line of demarcation between the wetted sediments and the unwetted sediments. As the wetness increases and the 'meniscuses overflow,' the next segment of soil volume becomes wetted provided that a supply of liquid is made available. If the supply of liquid is small in comparison to these transport processes, then the rate of movement of the wetted front will be solely dependent upon that supply.
>
> "While the movement and distribution of moisture can be modeled by diffusion theory, the controlling parameter is the source term, that is, the tank leak rate."

Assumption 3 is predicated on a soil moisture displacement model whereby existing soil moisture is displaced by that from a tank leak. The average soil moisture content was used.

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*Isaacson's comments on pages 15 through 17 are also pertinent to the basis for the validity of assumption 3.

Validity of Assumption

In addition to his comments on the basis of assumption 3, Isaacson also remarked on the validity of assumption 3 as follows:

"The principal concern with this assumption is whether the incremental moisture as a consequence of a tank leak is a displacement process, an additive process, or a combination of both. Several models can be hypothesized for very slow leak rate conditions. For example, in very dry soils the process may be additive; that is, moisture is retained up to a fixed volume percent within the pore spaces by capillary and the front advances with very little or no mixing or displacement. In soils where moisture has been added previously and allowed to redistribute, the process may be primarily that of displacement; and, in soils that contain more than about 14 vol% moisture, the process may be additive and displacement with mixing and diffusion taking place. A search of the literature would provide more understanding of these processes.

"Once a certain degree of saturation occurs (considerably less than total saturation), the capillary potential causes the liquid front to advance. The volume percent moisture that is retained in the soil as the liquid front advances is related to the dimensions of the pores between adjacent soil particles, surface tension of the liquid, viscosity, and osmotic effects. The osmotic effects are related to concentrations and types of dissolved salts in the liquid as well as soil properties.

"In order to establish the 'equilibrium' soil moisture content attendant to a tank leak, the properties enumerated above should be determined for each dry well, tank, or tank farm. In lieu of such characterization, empirical 'equilibrium' moisture contents could be determined based on experiments and empirical results. Lacking this, subjective judgment was used to arrive at the value of 3%."

Review Committee Findings

The Review Committee unanimously accepted assumption 3 as reasonable and valid.

ASSUMPTION 4. THE MAXIMUM TANK LEAK RATE $\underset{\sim}{s}$ IS 0.03 GAL/MIN

Concerning this assumption, Isaacson and Gasper stated, "The dry well response time is inversely proportional to the value of s chosen. The most probable leak rate will be significantly <0.03 gal/min because the tank-deactivation program has reduced the drainable liquid. Further reduction in volume will take place as a result of the planned jet pumping program."

Need/Basis for Assumption

The term s in the dry well response equation is the rate (ft^3/d) at which liquid leaks from a single-shell tank. The frequency of monitoring of dry wells is approximately inversely proportional to the assumed maximum tank leak rate.

The maximum tank leak rate depends upon several factors: nature and amounts of tank contents (liquid, salt cake, and sludge); size of hole in the tank; soil properties (i.e., hydraulic conductivity, permeability, etc.); liquid pressure head; and flux conditions (i.e., steady or nonsteady state) of leak. Review of confirmed tank leaks at Hanford indicates that, of all these factors, the size of the hole in the tank is dominant and in fact controls the leak rate. The high matric potential (soil suction) of the dry Hanford soil causes the liquid front to move at a rate dependent upon the volumetric relationships between the volume of waste that has leaked and the volume of soil that is wetted.

Historic leak rate data for previous known and suspected tank leaks* were analyzed statistically to determine the source term s to be used in the dry well response equation. This analysis led to the following confidence statement: 95%/95% Tolerance Interval: We are 95% confident that at least 95% of the population of single-shell tanks that will leak will have a leak rate <0.03 gal/min.

Validity of Assumption

Discussion of the Review Committee about the validity of assumption 4 centered on the answers to three questions:

- As compared to other factors, does the size of the hole in a tank control the leak rate?
- Is statistical analysis of leak rate data from previously known or suspected leaking tanks a suitable way to determine a maximum tank leak rate?
- Is an assumed maximum leak rate of 0.03 gal/min a conservative value?

*Data for Tanks 241-T-106 and 241-SX-110 were not used in the statistical analysis since it is known that these tanks were subjected to unusual conditions (e.g., sudden operational stresses, corrosion conditions, as well as structural and construction inadequacies).

The Review Committee, in the light of its previous extensive discussions of the hydrological properties of Hanford soils performed in considering assumptions 2 and 3, had little difficulty in concurring with Isaacson that the size of a hole in a tank does indeed control the rate at which liquid leaks from a tank. The Review Committee also readily agreed that a reasonable way to determine a maximum tank leak rate was to conduct a statistical analysis of data from previously confirmed or suspected leaking tanks.

The Review Committee noted that leak rate data for Tanks 241-T-106 and 241-SX-110 were deliberately excluded from the statistical analysis. Rates of leaks from these two tanks (1.7 and 0.2 gal/min, respectively) were considerably higher than those from any other known or suspected leaking tank. Inclusion of data for Tanks 241-T-106 and 241-SX-110 in the statistical treatment would have led to a calculated maximum tank leak rate, *s*, considerably higher than 0.03 gal/min and, correspondingly then, to increased frequency of monitoring all dry wells.

The Review Committee acknowledged that historical data for Tanks 241-T-106 and 241-SX-110 indicate that these two tanks were subjected to unusual conditions (e.g., sudden operational stresses, corrosion conditions, structural and construction inadequacies, etc.). There is thus probably sufficient reason to exclude leak rate data for Tanks 241-T-106 and 241-SX-110 from the population of data used in the statistical analysis to determine the maximum probable leak rate. An important consequence of not including data for Tanks 241-T-106 and 241-SX-110 in the statistical analysis is that the dry well response equation does not apply to and cannot be used to set a monitoring schedule for any other single-shell tanks that may also be subjected to unusual conditions. The Review Committee is pleased to note that this latter caveat has been clearly called out in the final published version of the Isaacson-Gasper report. In their published report, Isaacson and Gasper also say, "Currently, all of the single-shell tanks are inactive, except for removal of any remaining supernatant plus the interstitial liquids. It is unlikely that these tanks will be subjected to sudden stress conditions."

Review Committee Findings

With proper acknowledgement in the Isaacson-Gasper report of the implication of excluding leak rate data for Tanks 241-T-106 and 241-SX-110 from the statistical analysis, the Review Committee accepts assumption 4.

ASSUMPTION 5. RADIOACTIVE RUTHENIUM IS A SATISFACTORY WASTE FRONT TRACER

In their report, Isaacson and Gasper (1981) state, "As described previously, the tenth-value thickness, $l_{\bar{m}}$, is dependent logarithmically on ruthenium concentration. This sluggish dependence minimizes the impact of choice of ruthenium concentration. However, the ruthenium concentrations vary significantly from tank to tank with a current range of concentration being 2.4 x 10^{-5} to 2.4 x 10^{-2} Ci/L. Using the ISOSHLD computer shielding model for tenth-value thickness with these concentrations leads to a range of tenth-value thicknesses from 12.6 to 20.4 cm." The tenth-value thickness of 12.6 cm applies to the oldest wastes having low ruthenium concentrations.

Need/Basis for Assumption

For the dry well monitoring equipment to function at all, obviously some gamma-emitting radionuclide or radionuclides must migrate through the soil toward the dry well. There is much evidence to back up the contention that radioactive ruthenium is very mobile in Hanford soils and is, indeed, more mobile than any other radionuclide. The results of Cowser and Parker (1958), Spitsyn et al. (1958), Brown et al. (1955, 1958), and Routson et al. (1979) are all relevant.

In particular, Brown et al. (1955, 1958) studied migration of radionuclides when actual low-level liquid waste containing 80 g/L of various inorganic salts was added to a Hanford crib; ^{106}Ru moved downward at a much faster rate than any other radionuclide present in the waste. Brown and colleagues in various laboratory-scale tests of the migration of radionuclides in Hanford soil columns also noted the increased mobility of radioactive ruthenium over that of other radionuclides. Radioactive ruthenium in Hanford wastes is believed to be present as anionic and/or neutral species which are not sorbed by various sediments or soils.

Furthermore, the distribution of radionuclides in Hanford soil is controlled to a major extent by the sorption and unsaturated flow characteristics of water in the Hanford soils. At the leading edge of a leak plume, the concentration of lesser-sorbed radionuclides increases relative to those of better sorbed radionuclides. The ratio of concentrations of nonsorbed-to-sorbed activities increases with increasing lateral distance or depth of the leak. Ruthenium-106 is poorly sorbed compared with ^{137}Cs; at the leading site of the leak plume the ^{106}Ru/^{137}Cs concentration ratio is highest.

Validity of the Assumption

The Review Committee generally did not have any great difficulty in accepting the physical evidence that radioactive ruthenium migrates faster through Hanford soils than any other radionuclide present in liquid wastes stored in the single-shell tanks. However, several members of the Review Committee noted:

- Cobalt-60 ($t_{1/2}$ = ~5 yr) is present in most of the single-shell tank waste; although not nearly as mobile as ^{106}Ru, ^{60}Co also is known to migrate through Hanford soils at rates faster than many other radionuclides (e.g., ^{137}Cs).

- Monitoring equipment used in the dry wells is nonspecific and detects radiation from any gamma-emitting radionuclides (e.g., ^{106}Ru-^{106}Rh, ^{60}Co, etc.) that approach the well.

- The dry well response equation is based partly upon ISOSHLD computer code calculations of tenth-value soil thickness, taking into account as key input data only the decay characteristics of ^{106}Ru-^{106}Rh. Results of these calculations will be in error to the extent that dry well monitors detect gamma energy from radionuclides other than ^{106}Ru-^{106}Rh and that the tenth-value thickness would be significantly different under these conditions.

- The tenth-value thickness is both a function of gamma energy and concentration of the predominant radionuclides. As the concentration decreases, the contribution of the low-energy components resulting from Brehmsstrahlung increases. Since ^{60}Co is present in very low concentrations, the response characteristics of the gamma detectors will not change significantly. This is generally true for old wastes regardless of which radionuclides are present.

<u>Review Committee Findings</u>

The majority of the Review Committee, although acknowledging ^{60}Co mobility and its possible detection by dry well monitoring equipment, still regard assumption 5 as perfectly valid.

ASSUMPTION 6. LEAK OCCURS ON PERIMETER OF TANK AT POINT ABOUT 12 FT ON EITHER SIDE OF NEAREST DISTANCE TO A DRY WELL

Concerning this assumption, Isaacson and Gasper state:

"...12 ft is 5% of the tank perimeter. The response time is determined using a distance, b, to this point. If a 0.03-gal/min leak were to occur at any point on the tank within 6.2 ft of a dry well in 200 West Area and within 5 ft of a dry well in the 200 East Area, the count rate could increase to action level before being detected. If the leak occurred farther than 12 ft away, more monitoring would be performed than would be necessary to assure that the leak would be detected before the count rate reached the action level."

<u>Need/Basis for Assumption</u>

This assumption is different than assumptions 1 through 5 because it relates primarily to criteria for minimum frequency of dry well monitoring (and associated risk analyses); assumption 6 is not required for derivation of the empirical dry well response equation. On the basis of its charter, the Review Committee did not evaluate or judge assumptions (assumption 6) dealing with risks or attempts to relate dry well monitoring to leak location.

Even though the Review Committee did not formally review assumption 6, one member (R. E. Isaacson) provided the following comments:

> "Mathematical analysis continues to investigate assuming a greater distance than 5% (12 ft). The impact is to decrease the monitoring frequency significantly for dry wells near the tanks and less significantly for dry wells further from the tanks. Since the mathematical basis for selecting larger percentages of the perimeter is still under investigation, the 5% distance is utilized for determining monitoring frequencies.
>
> "Ninety percent of the leaks will occur outside the range of the dry well response time for a given dry well based on this assumption. Furthermore, this assumption is based upon the probable maximum leak rate rather than the most probable or mean leak rate. Based on the more recent value of 0.03 gal/min as the maximum leak rate, and mean leak rate of 0.0115 gal/min, 50% of the leaks will have response times of >3 wk in 200 West Area and >6 wk in 200 East Area.
>
> "The monitoring frequencies thus calculated tend to be very conservative and much less than 5% of all possible leaks will exceed the alert level between monitoring periods.
>
> "Assuming that there is equal probability of a leak occurring within any 1 ft segment of a tank and that one leak will occur each week, then the probability of a leak occurring is 4.56×10^{-5}/ft of tank perimeter per week. There are 101 wells to be monitored once each week based on a maximum leak rate of 0.03 gal/min, thus the probability of a leak exceeding the alert level is $101 \times 4.56 \times 10^{-5}$ or 0.46%. In other words, based on the proposed monitoring shedule, there is 99.5% probability of detecting a leak before the count rate exceeds 160 counts/sec above background."

Validity of Assumption

The Review Committee did not discuss the validity of assumption 6.

Findings of Review Committee

The Review Committee did not come to any formal conclusion regarding the basis or validity of assumption 6.

ALTERNATIVE APPROACHES TO A DRY WELL RESPONSE EQUATION

By its charter, the Review Committee was committed to "consider, develop, and evaluate alternative approaches to derivation of a dry well response equation." Two alternative approaches to the empirical approach used by Isaacson were investigated: analytical models based on diffusion theory and computer models.

ANALYTICAL MODELS

By solving the partial differential equations (diffusion equations) governing the motion of fluids through soils, H. A. Forrester of the Review Committee derived an analytical model (diffusion model) of the motion of the plume from a leaking Hanford waste tank. (Complete mathematical details of Forrester's elegant derivation are presented in Appendix B of this report.) Although originally intended as a replacement for the Isaacson empirical model, the diffusion model produces results essentially equivalent to those yielded by the Isaacson model. The Review Committee believes that more adequate verification of the basis, assumptions, and derivation of the dry well response equation neither can be found nor is required.

COMPUTER MODELS

Computer codes that model moisture flow in unsaturated sediments such as those in Hanford tank farms are currently available. As mentioned earlier, Lu (1980) has reported results of computer-assisted modeling of the transport of liquid and radionuclides from the 241-T-106 Tank leak through vadose zone soils. From this background, it is easy to generalize that computer modeling techniques can and should be used to develop a frequency schedule for monitoring dry wells around single-shell tanks. Indeed, one external reviewer (Reviewer 3) was particularly keen on the supposed advantages of computer modeling of tank leaks over the Isaacson geometric approach.

Before the availability of the full diffusion model (Appendix A), the Review Committee spent much time debating the merits of computer modeling of tank leaks versus the empirical approach taken by Isaacson in deriving the dry well response equation. Knowledgeable and experienced Review Committee members advised that computer solutions to the moisture and convective dispersion equations for unsaturated sediment systems require parameters that simply are not available for the Hanford tank farm sediments. Acquisition of the required parametric data requires a great expenditure of time, money, and effort. Hence, the Review Committee concluded that application of computer modeling techniques to develop tank-by-tank dry well monitoring schedules just was not practical. Of course, the later advent of the diffusion model provided robust support for the judgment to reject computer modeling in favor of Isaacson's empirical equation.

ACKNOWLEDGMENTS

During its tenure and deliberations, the Review Committee received much valuable assistance from several sources. The Committee desires particularly to acknowledge the important contributions of the external reviewers for their time and effort in providing objective and authoritative reviews and for suggestions for improving the response equation's derivation and presentations; Andrea Talbot for expert and unflagging secretarial help in setting up Review Committee meeting times and facilities, taping meeting proceedings, transcribing meeting proceedings, etc; and Fred Stong (Rockwell's Instrument & Electrical Design & Development Unit, Development Engineering Group) for providing the Review Committee with detailed information on equipment and systems used in monitoring dry wells. Finally, the Review Committee Chairman (W. W. Schulz) thanks the Review Committee members for their cooperation, patience, and understanding in successfully completing the Review Committee's work.

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APPENDICES

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APPENDIX A
ABSTRACTS OF COMMENTS OF EXTERNAL REVIEWERS

Pertinent comments and criticisms of the three external reviewers who reviewed all or parts of an early draft of RHO-ST-34 are collected in this appendix.* The action taken in response to each of the issues or points raised by the external reviewers is also briefly indicated. Note that page numbers cited in external reviewer's comments refer to an early draft of RHO-ST-34 while page numbers cited in Responses refer to page numbers in the final published version of RHO-ST-34.

REVIEWER 1

Comment One

"I was surprised to read that this report primarily discusses monitoring for single-shell tanks, since I was of the impression that they have all been taken out of service."

Response

Other reviewers, both in-house and external, made essentially this same observation. Accordingly, the text was revised (p. 4) to note that interstitial liquids that cannot be drained by normal pumping methods are still present in single-shell tanks.

Comment Two

"The comments on page 9 discuss c as a constant detector calibration factor which is not strictly true. This calibration factor will change due to relative source size changes. If the calibration factor which is used for a trigger case is considered, it would be conservative for all other cases."

Response

In considering this comment, the Review Committee came to realize, thanks to much helpful input from Mr. Fred Stong, that the external reviewer did not fully understand how the gross count scintillation probes used in monitoring dry wells actually operated. To remedy this deficiency, new text (pp. 10-11) was added in the final report to describe the various dry well detection systems. Mr. Stong also noted that the gross scintillation probes were compared twice a day with standard ^{137}Cs

*Isaacson, R. E. and K. A. Gasper (1981), A Scientific Basis for Establishing Dry Well-Monitoring Frequencies, RHO-ST-34, Rockwell Hanford Operations, Richland, Washington.

sources; the term c is simply the slope of a logarithmic plot of dose rate versus count rate as determined by such comparisons. A definition of c was provided in the final published report.

Comment Three

"The paragraph at the bottom of page 11 is quite confusing. As a waste front approaches, the contribution from high-energy photons is always increasing simply because we are adding more material in the view of the detector. The low-energy contribution is dominated by scattered high-energy photons and, hence, increases directly proportional to the high-energy contributions. The last paragraph on this page, however, is written such that it seems to imply the high-energy dose rate contribution does not continually increase as the low-energy end is enhanced. This is not true. I believe I understand what the authors are indicating, but it is written in a very confusing fashion."

Response

The Review Committee did not judge this criticism to be of any particular significance; the paragraph mentioned was included unchanged in the final report (p. 21).

Comment Four

"On page 19 the calibration constant c for the scintillation probe is indicated at 0.918. Some discussion needs to be made of this efficiency and calibration constant. It looks too high to be typical of a scintillator. Thus, I believe some specific description and discussion must be added."

Response

See response to comment 2.

Comment Five

"Pages 24 and 25 are a series of assumptions to which should be added how a well is logged and with what type scintillator. Some consideration should be given to a discussion of background and thus a basic statistical criteria in terms of any trigger or response level."

Response

The Review Committee rejected the idea of including as a basic assumption "how a well is logged and with what type scintillator." The final report, however, does contain a section (pp. 10-12) that describes the various detectors used to monitor dry wells.

Comment Six

"Somewhere in the report was a discussion of bulk density which was listed at 1.83. I believe the bulk density of Hanford soils is from 1.41 to 1.6."

Response

Several knowledgeable members of the Review Committee confirmed that the bulk density of Hanford soil is indeed about 1.8.

Comment Seven

"The typical soil moisture contents that I am aware of are on the order of 4%."

Response

The Review Committee agrees that Hanford soils contain typically 4 wt% moisture or, for a soil density of 1.83, about 8 vol% (assumption 3).

Comment Eight

"The way the report discusses tenth-value thickness is confusing since a tenth-value thickness is a constant, not a variable for a given energy and matrix."

Response

This particular comment was picked up by several of the Review Committee members as reflecting their own concern that the original draft of RHO-ST-34 did not adequately discuss tenth-value thicknesses. In response to this concern the final report contains (pp. 18-20) an extended discussion of tenth-value thicknesses and their use.

Comment Nine

"One final comment. I believe a discussion to be in order for the various detection techniques which one might use in monitoring wells. This would be a discussion of in-well monitoring capability and the specific form in which it should be used."

Response

The Review Committee unanimously agreed; the requested discussion appears on pages 10 and 12 of the final report.

REVIEWER 2

Comment One

"Are the assumptions that permit the formulation of the dry well equation so arbitrary and so undetermined as to not contribute much to reality? I wonder if the mathematical presentation and equation formulation present an air of knowledge and exactitude that is not warranted. Some features of the formulation that trouble me are:

1. Need for variable volume fraction of soil moisture content

2. Leak rate factors such as hole size, waste head, and location of leak relative to dry wells

3. Soil permeability

4. Ruthenium concentrations

5. Gravitational effects for downward as well as outward movement of liquid waste."

Response

The Internal Review Committe was convened and chartered exactly to address the issues raised by Reviewer 2 concerning the basis and validity of the key assumptions underlying the empirically-derived dry well response equation. As noted in the body of this report, the Review Committee found these assumptions to be valid.

Comment Two

"It would be easier for me, I think, to build a schedule for dry well sampling based on past experience and observation data. Why not set up the system purely empirically and not try to make such a complex calculation. Past experience seems extremely valuable to me."

Response

The Review Committee concurs that it is indeed possible to use past experience and observation data to set a schedule for monitoring dry wells. A much more credible monitoring schedule can be set, however, by application of a valid dry well response equation.

Comment Three

"It would be helpful to clearly state the purpose of the dry well program. Is it:

a. A primary leak detector system

b. A backup or secondary leak detector backup system

c. A location of leak system

d. Does it serve some other purpose?"

Response

Additional information was added in the Introduction and in Section 1.3 of the final report to respond to this comment.

Comment Four

"I have some difficulty seeing how the wells can be used for intrusion monitoring as stated on page 2."

Response

The final report points out that it is the remote possibility of intrusion of water into tanks, not people, which is also of concern in dry well monitoring.

Comment Five

"The use and definition of the tenth-value layer as formulated on pages 11 and 12 causes me some difficulty. Particularly, a tenth-value layer is a percentage reduction quantity and not a function of the amount of radiation or the amount of Ru present. The formulation of this part of the presentation needs review and clarification."

Response

The Review Committee was in complete accord with this comment. The final report contains (Section 2.2) adequate and complete explanation of tenth-value thicknesses and their use in deriving the dry well response equation.

Comment Six

"The illustration on page 6 (Figure 1) needs to be redrawn to assure dimensions a and b are more representatively illustrated."

Response

Correction made in final report. See Figure 2.

Commment Seven

"I have some difficulty understanding the soil moisture volume fraction presentation as developed on pages 5, 19, and 24. Does this presentation claim the soil moisture volume fraction is 0.08 before a leak and would not significantly increase, or is this the value after the leak? It seems surprising the value would stay so low. If it stays so low, why does not a leaked water volume move rather rapidly to the groundwater? A few sentences of explanation would be helpful to me."

Response

The final report was modified to include the explanation requested by the reviewer. This report presents (pp. 21-25) a detailed examination and review of assumption 3.

Comment Eight

"Equation 13 on page 9 is introduced with no derivation. Some additional explanation would be helpful."

Response

This comment refers to Equation 13 in the final report. As requested by the reviewer, additional text was added to explain the origin of Equation 13.

Comment Nine

"Some rewording to avoid confusion may be necessary, especially with regard to the first paragraph in the executive summary. This reads as if single-shell tanks still contain liquid high-level radioactive waste. A better presentation on their 'liquid' contents is needed."

Response

The final report does not have an executive summary; words that state the present content of the single-shell tanks are included in the Introduction.

REVIEWER 3

Comment One

"The subject of dry well monitoring criteria has been avoided for too long. The importance of the subject is only equaled by its difficulty. This report is the first and only serious attempt to address this issue that I have seen. Mr. Isaacson and Mr. Gasper are to be complimented for their perception of the problem and for their boldness of examination. In my opinion, the approach described shows a great deal of creativity and therefore has much to offer.

"I do have serious concerns that this study will be the final word on the subject. The study is highly empirical in nature and clearly demonstrates that our understanding of the tank leak process and subsequent transport of radioactivity is not very sophisticated. The current study will be most useful if it is used as a conceptualization of the problem and an initial guide to analysis. I think the concept of the incremental moisture content has merit and the geometric analysis of plume propagation is as good as the available information allows."

Response

The comments of Reviewer 3 were made before Dr. Forrester had completed his detailed, sophisticated mathematical analysis. As noted previously, the diffusion model developed by Forrester produces results essentially equivalent to those yielded by the Isaacson model. In retrospect, Reviewer 3 was correct in stating that the current study (i.e., Isaacson's equation) will be most useful if it is used as a conceptualization of the problem and an initial guide to analysis (i.e., diffusion model).

<u>Commment Two</u>

"I feel very strongly that the importance of the problem requires better analysis. I think computer modeling is both practical and desirable as well as expensive. Much progress has been made in our ability to put bounds on our parameter estimates as well as our basic modeling skills. A Rockwell model is reported to have simulated the T-106 leak to a reasonable degree and using the same input parameters should give more defensible results than those presented here."

<u>Response</u>

Again, this comment was made before Dr. Forrester showed that an elegant mathematical model (diffusion model, see Appendix B) gave results essentially equivalent to Isaacson's equation. The Review Committee agrees that computer modeling is expensive, but in view of the corroborating diffusion model, finds that it is neither desirable nor necessary.

<u>Comment Three</u>

"The geometric analysis of the plume propagation is appropriate as a first step. This idea is based on a qualitative analysis of the T-106 Tank leak. It is an empirical result that should be used with some caution because it cannot be fully explained. It would be possible to use this in some way to verify computer modeling efforts. The main flaw, as I see it, is the 'assumption' of constant moisture content throughout the plume. This implies instantaneous propagation of input signals. This 'action at a distance' is fundamentally impossible in an unsaturated soil system. We should not say that we are <u>assuming</u> that it happens. I think what we mean is that the empirical evidence suggests that the response times are small compared to the travel times and that to a first approximation we use this type of analysis. Perhaps we are splitting hairs, but I think there is an important difference between an assumption and an approximation. The empirical nature of the geometric analysis should be emphasized and more supporting evidence should be provided. A result not justifiable on theoretical grounds must be backed up with a preponderance of empirical evidence."

<u>Response</u>

The final version of RHO-ST-34 does clearly indicate the nature of the geometric approach taken by Isaacson to derive the dry well response equation. The last sentence in this comment of Reviewer 3 is not appropriate since the advent of the diffusion model. The Review Committee also rejected as inappropriate the judgment of Reviewer 3 regarding use of the results of empirical geometric analysis with computer modeling. Several Review Committee members agreed with the contents of Reviewer 3 regarding "assumptions" versus "approximations."

Comment Four

"Validity of Assumptions Made in Deriving the Response Equation."

Assumption One. The spherical geometry may be the best "simple" way to analyze the data without using computer models. My comments above relate my misgivings of how the concept is presented.

Assumption Two. This is a physical impossibility and should be identified as such. Nevertheless, it does look like a reasonable approximation from the limited evidence available.

Assumption Three. Acceptable.

Assumption Four. Acceptable only because 0.044 gal/min is the largest leak rate experienced to date.

Assumption Five. No comment.

Assumption Six. Acceptable.

Response

No response required.

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APPENDIX B
ANALYTICAL MODEL OF LEAK PLUMES

INTRODUCTION

An analytical model of the plume produced by a leak in a waste tank was derived by solving the partial differential equations governing motion of fluid in soil. Some simplifications are introduced, but the essential physics of the situation are covered. Since the differential equations are generally called diffusion equations, the analytical model is termed the diffusion model.

The diffusion model originally was intended as an alternative to the model observed by Isaacson. Somewhat unexpectedly, the diffusion model turned out to be in essential agreement with the Isaacson model. It is, of course, of great value to be able to support the Isaacson model on theoretical grounds.

DERIVATION OF DIFFUSION MODEL

Notation

U = Volume concentration of fluid

U_0 = Concentration of fluid in soil prior to the leak

U_1 = Critical concentration of fluid

V = Volume concentration of tracer radioactive element

A = V/U = ratio of tracer to fluid

A_0 = Value of A in leaking fluid

K = Permeability of soil (i.e., diffusion constant)

$$K = \begin{cases} K_1 & \text{if } U < U_1 \\ K_2 & \text{if } U \geq U_1 \end{cases}$$

r = Distance from leak source

t = Time, $t = 0$ is start of leak

$X_1, X_2, \ldots, X_n$ = Coordinates centered at leak

n = "Dimension" of leak, where

$n = 2$: leak confined between impermeable strata

$n = 3$: unconfined leak

$R(t)$ = Radius of leak front at time t

R_0 = Limiting value of $R(t)$ (for $n = 3$)

S = Rate of leak

H = Height of confined layer (for $n = 2$).

Leak Equations

1. $\frac{\partial U}{\partial t} = K \frac{\partial^2 U}{\partial r^2} + \frac{n-1}{r} \frac{\partial U}{\partial r}$

2. $U\frac{\partial A}{\partial t} = K \frac{\partial U}{\partial r} \frac{\partial A}{\partial r}$

3. $U = U_0$ for $t \leq 0$

4. $A = A_0$ for $r = 0$ and $t \geq 0$

5. The rate of U at $r = 0$ for $t = 0$ is: $\begin{cases} S \text{ if } n = 3 \\ S/H \text{ if } n = 2 \end{cases}$

   Here $A = A_0\, F(\beta)$ where $F(\beta) = \begin{cases} 0 \text{ if } \beta > 0 \\ 1 \text{ if } \beta < 0 \end{cases}$

   and $\beta = \int r^{n-1}\, U\, dr$.

Synopsis of Results

As the leak fluid moves through the soil, it displaces fluid already in the soil. The displaced fluid undergoes surprisingly little mixing with the leak fluid. Thus, the forward part of the leak is formed by fluid with little or no tracer element present. In the region between the leak and the displaced fluid, both A and U are nearly constant except for the immediate vicinity of the leak. This conclusion is essentially the same as that given by the Isaacson model.

There is an exceptional case (i.e., when $U_0 = 0$) when the Isaacson model does not apply. However, since the soils in the tank farms at Hanford have U_0 in the range of 0.04 to 0.12, U_0 is a substantial fraction of U_1 (which is about 0.10). Therefore, this case is not considered relevant.

DETAILS OF ANALYTICAL MODEL

Derivation

Two matters must be considered in deriving the equations governing the formation of the plume of a leak: (1) the motion of fluid through the soil and (2) the motion of a tracer element carried by the fluid. It is assumed that there is a radionuclide (tracer) in the tank fluid that moves without being absorbed by the soil. It is the detection of this radionuclide at a dry well that permits detection of the leak. If the soil were absolutely dry, the motion of the fluid and of the tracer radionuclide would coincide. However, the amount of groundwater in even relatively dry soils will be enough to have a substantial effect.

Let U denote the volume concentration of fluid and V denote the volume concentration of the tracer. U and V both are dimensionless, i.e., they have units of the form volume/volume. Let A = V/U denote the fraction of fluid volume formed by the tracer. Thus, A is also a dimensionless number.

Let $X_1, X_2, \ldots, X_n$ denote rectangular coordinates centered at the leak. The reason for choosing an arbitrary dimension n is economy since two important cases occur. The case n = 3 corresponds to the leak being unconstrained, and n = 2 corresponds to the leak being confined between impermeable layers.

Let $F_1, F_2, \ldots, F_n$ be the rates of diffusion of the fluid in the $X_1, X_2, \ldots, X_n$ direction. It is assumed that the rate of motion of the fluid is proportional to the rate of change of concentration and that the fluid moves from higher to lower concentrations. Consequently,

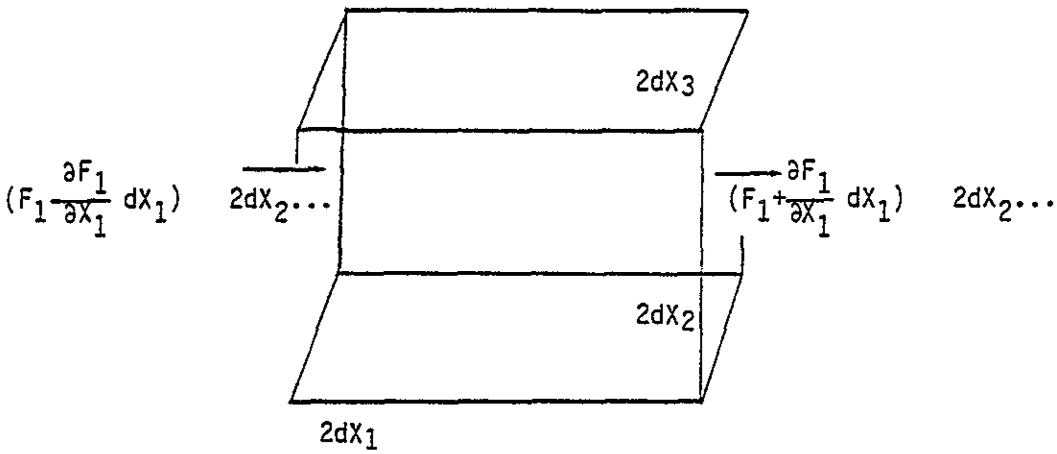
$$F_i = -K \frac{\partial U}{\partial X_i} \tag{1}$$

where K is the diffusion constant.

Let $E_1, E_2, \ldots$ denote the rates of diffusion of the tracer element. The assumption that the tracer element moves with the fluid is expressed by:

$$E_i = AF_i = -K A\frac{\partial U}{\partial X_i} \tag{2}$$

where A = V/U, the fraction of the tracer present in the fluid. Consider a rectangular box of sides $2dX_1, 2dX_2, \ldots$ centered at $(X_1, X_2, \ldots)$.



The flow into the box across the face at $X_1 - dX_1$ is

$$(F_1 - \frac{\partial F_1}{\partial X_1} dX_1) \times (\text{area of face}) \tag{3}$$

while the flow out of the box across the face at $X_1 + dX_1$ is

$$(F_1 + \frac{\partial F_1}{\partial X_1} dX_1) \times (\text{area of face}) \tag{4}$$

Thus the net accumulation in the box due to flow in the X_1 direction is

$$-\frac{\partial F_1}{\partial X_1} \times 2dX_1 \times \ldots \times 2dX_n \tag{5}$$

The total rate of accumulation per unit volume of soil is the sum of all such terms divided by the volume ($2dX_1 \times \ldots \times 2dX_n$) of the box. That is,

$$\frac{\partial U}{\partial t} = - \Sigma_{r=1}^{n} \frac{\partial F_r}{\partial X_r} = \Sigma_{r=1}^{n} \frac{\partial}{\partial X_r} (K \frac{\partial V}{\partial X_r}) \tag{6}$$

A similar argument yields

$$\frac{\partial V}{\partial t} = -\Sigma_{r=1}^{n} \frac{\partial F_i}{\partial X_r} = \Sigma_{r=1}^{n} \frac{\partial}{\partial X_r} (KA \frac{\partial U}{\partial X_r}) \tag{7}$$

or, since V = AU,

$$\frac{\partial AU}{\partial t} = \Sigma_{r=1}^{n} \frac{\partial}{\partial X_r} (KA \frac{\partial U}{\partial X_r}) \tag{8}$$

Subtraction from (5) of A times the equation for U gives

$$U \frac{\partial A}{\partial t} = \Sigma_{r=1}^{n} \; K \frac{\partial U}{\partial X_r} \frac{\partial A}{\partial X_r} \tag{9}$$

There is a criticism that can be made. One is that possible anisotropies of the soil have not been taken into account. The equations allowing for anisotropy are:

$$\frac{\partial U}{\partial t} = \Sigma_{r=1}^{n} \Sigma_{j=1}^{n} \frac{\partial}{\partial X_i} (K_{ij} \frac{\partial U}{\partial X_j}) \tag{10}$$

$$U \frac{\partial A}{\partial t} = \Sigma_{r=1}^{n} \Sigma_{j=1}^{n} K_{ij} \frac{\partial U}{\partial X_j} \frac{\partial A}{\partial X_i} \tag{11}$$

A change of coordinates will reduce these equations to the previous form, and the change of coordinates can be so chosen as not to change the scale in any particular direction of interest, i.e., along a line from the leak to the dry well.

The diffusion "constant" K will be taken as being a function of U and as having the form

$$K = \begin{cases} K_1 \text{ if } U < U_1 \\ K_2 \text{ if } U \geqq U_1 \end{cases} \tag{12}$$

This is an approximation to the physical situation. Fluid moves slowly in relatively dry soil and much faster at a certain level of ground moisture represented by U_1. It is $\sim U_1 = 0.10$. The corresponding values of K are denoted by K_1 and K_2; K_1 is $\sim 10^{-5}$ cm^2/sec, and K_2 is $\sim 10^{-2}/sec$. That is, K_2 is $\sim 1{,}000$ times K_1.

Let r denote $\sqrt{X_1^2 + X_2^2 + . . . +, X_n^2}$ so that r is the distance from the leak. Hereafter, U and A will be taken as functions of r and of t = time. Now

$$\frac{\partial}{\partial X_i} = \frac{X_i}{r} \frac{\partial}{\partial r}$$

so the equations for U and A become:

$$\frac{\partial U}{\partial t} = K\left(\frac{\partial^2 V}{\partial r^2} + \frac{n-1}{r} \frac{\partial U}{\partial r}\right) \tag{13}$$

$$\frac{U \partial A}{\partial t} = K \frac{\partial U}{\partial r} \frac{\partial A}{\partial r} \tag{14}$$

$$K = \begin{cases} K_1 \text{ for } U < U_1 \\ K_2 \text{ for } U \geqq U_1 \end{cases} \tag{15}$$

It is assumed that a constant level of ground moisture is present with a value of U_0 (i.e., $U = U_0$ for $t = 0$). The leak is assumed to start at time, t = 0, and have a constant flow rate. The expression for the leak rate takes two forms. For the case n = 3 (i.e., unconstrained plume), the leakage rate is simply S (with units volume/time). In the case n = 2, with the leak constrained by impermeable layers at a distance H apart, the leak rate is S/H (with units area/time, or volume/length/time).

Method for Solving Diffusion Equations

The method for solving the equations for U is first to obtain a solution U_{int} for

$$\frac{\partial U}{\partial t} = K_1\left(\frac{\partial^2 U}{\partial r^2} + \frac{n-1}{r} \frac{\partial U}{\partial r}\right) \tag{16}$$

for $r \leqq R(t)$, where $R(t)$ is the value of r for which

$$U_{int} = U_1 \tag{17}$$

Then obtain a solution U_{ext} of

$$\frac{\partial U}{\partial t} = K_2 \left(\frac{\partial^2 U}{\partial r^2} + \frac{n-1}{r}\frac{\partial U}{\partial r}\right) \tag{18}$$

for $r \geqq R(t)$, so as to satisfy the patching conditions when $r = R(t)$

$$U_{ext} = U_{int} \tag{19}$$

$$\frac{\partial U_{ext}}{\partial r} = \frac{\partial U_{int}}{\partial r} \tag{20}$$

The solution U is then the result of patching these two equations together. The location of $r = R(t)$ will be called the leak front.

In the case $n = 2$ (i.e., the plume confined between impermeable layers) there is a leak front only when the leak rate S is sufficiently large, i.e., when

$$S > 4\pi K_2 H(U_1 - U_0)$$

In this case, the plume will be well described by the Isaacson model. If the leak rate is small so that no leak front forms, an explicit solution is given by $U = U_{ext}$ for all r and t. In the case $n = 3$ (i.e., an unconfined plume), a leak front always exists. Moreover, R(t) will approach a limiting value R_0 which will be small. There will be substantial leakage across the leak front so that $U = U_{ext}$ will be the significant solution at a dry well. Expressions for U_{int}, U_{ext} are given later.

The solution of the equation for A is obtained by the method of characteristics. A first order equation $M\frac{\partial A}{\partial r} - N\frac{\partial A}{\partial t} = 0$ has the following features:

- If β is any nontrivial solution, then the general solution, A, is given by $A = F(\beta)$ for an arbitrary function F.

- The solution A is constant along the solution curves of $N\,dr + M\,dt = 0$. If

$$\frac{dr}{dt} = -\frac{M}{N}$$

then

$$\frac{dA}{dt} = \frac{\partial A}{\partial r}\frac{dr}{dt} + \frac{\partial A}{\partial t} = -\frac{M}{N}\frac{\partial A}{\partial r} + \frac{\partial A}{\partial t} = 0$$

- If $\beta(r,t) = 0$ is the solution of $N\,dr + M\,dt = 0$, then β is a solution of the partial differential equation.
- The equation $N\,dr + M\,dt = 0$ has a solution $\beta = \int N\,dr$ when the equation is exact, that is, when $\frac{\partial N}{\partial t} = \frac{\partial M}{\partial r}$.

Now

$$\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial U}{\partial r}\right) = r^{n-1}\frac{\partial^2 U}{\partial r^2} + (n-1)r^{n-2}\frac{\partial U}{\partial r} = r^{n-1}\left(\frac{\partial^2 U}{\partial r^2} + \frac{n-1}{r}\frac{\partial U}{\partial r}\right) \quad (21)$$

and so the differential equation for U can be written

$$\frac{\partial}{\partial t}\left(r^{n-1}U\right) = K\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial U}{\partial r}\right) = \frac{\partial}{\partial r}\left(K\,r^{n-1}\frac{\partial U}{\partial r}\right) \quad (22)$$

Thus $(r^{n-1}U)dr + (Kr^{n-1}\frac{\partial U}{\partial r})dt = 0$ is exact. Hence its solution, β, is a solution of $U\frac{\partial A}{\partial t} = K\frac{\partial U}{\partial r}\frac{\partial A}{\partial r}$ (after multiplying by r^{n-1}) and hence $\beta = \int r^{n-1}U\,dr$. Thus $A = F(\int r^{n-1}U\,dr)$ for some function F. The determination of F remains to be carried out.

COMPARISON OF DIFFUSION AND ISAACSON MODELS

The Isaacson model assumes that U (or perhaps V) is uniformly distributed inside a region $r = R(t)$, and that U (or V) is a constant value independent of time for $r \leq R(t)$. Thus for $n = 3$ (unconfined plume), $R(t) = \text{Const.}\sqrt[3]{t}$, and for $n = 2$ (confined plume), $R(t) = \text{Const.}\sqrt[2]{t}$.

<u>Solutions for the Confined Plume</u>

The solution of $\frac{\partial^2 U_{int}}{\partial r^2} = K_1 \left(\frac{\partial^2 U_{int}}{\partial r^2} + \frac{1}{r}\frac{\partial U_{int}}{\partial}\right)$ (in the case n = 2)

is $U_{int} = U_0 + S/(4\pi K_2 H) \exp(-r^2/4K_2 t)$. The leak front $r = R(t)$ is the solution, for r is a function of t, of $U_1 = U_0 + S/(4\pi K_2 H) \exp(-r^2/4K_2 t)$, which is $R(t) = \sqrt{\log \frac{S}{4\pi K_2 H(U_1 - U_0)}} \cdot \sqrt{4K_2 t}$. This is valid only for $\log \frac{S}{4\ K_2 H(U_1 - U_0)} > 0$, that is, for $S > 4\pi K_2 H(U_1 - U_0)$. If $S < 4\pi K_2 H(U_1 - U_0)$, the solution is $U = U_{int} = U_0 + S/(4\pi K_1 H) \exp(-r^2/4K_1 t)$. If $S > 4\pi K_2 H(U_1 - U_0)$, the exterior solution, U_{ext}, is determined as follows: The first step is to determine the amount of leak fluid contained behind the leak front. The total amount of fluid leaked is St at time t, and the amount behind the front is the integral over $r \leq R(t)$ of $(U - U_0)$ times the element of volume $H \cdot rdr \cdot d\theta$ (in polar coordinates). The amount is:

$$\frac{S}{4K_1 H\pi} \cdot H \cdot 2\pi \int_0^{RH} r\, e^{-r^2/4K_1 t} dr \tag{23}$$

$$= St\ (1 - e^{-R^2(H)/4K_1 t}) \tag{24}$$

$$= St\ (1 - e^{-\log(S/4\pi K_1 H(U_1 - U_0))}) \tag{25}$$

$$= St\ \left(1 - \frac{4\pi K_1 H(U_1 - U_0)}{S}\right) \tag{26}$$

Now $\frac{4\pi K_2 H(U_1 - U_0)}{S}$ is < 1, so a constant fraction

$$St\left(1 - \frac{4\pi K_2 H(U_1 - U_0)}{S}\right) \tag{27}$$

of the fluid moves outside the leak front $r = R(t)$. Taking U_{ext} in the form

$$U_{ext} = U_0 + \frac{\sigma}{4\pi K_1 H} \; e^{-r^{2}/4K_1 t} \qquad (28)$$

for some constant σ, and such that

$$U_{ext} = U_1 \text{ at } r = R(t) \qquad (29)$$

gives

$$U_1 - U_0 = \frac{\sigma}{4\pi K_2 H} \; e^{-\left(\frac{K_2}{K_1} \cdot \frac{(RH)^2}{4\pi K_2 t}\right)} \qquad (30)$$

$$= \frac{\sigma}{4\pi K_2 H} \cdot \left(\frac{4\pi K_2 H(U_1 - U_0)}{S}\right)^{\frac{K_2}{K_1}} \qquad (31)$$

so that

$$\sigma = \left(\frac{S}{4\pi K_2 H(U_1 - U_0)}\right)^{\left(\frac{K_2}{K_1} - 1\right)} \cdot S \qquad (32)$$

Since

$$\frac{S}{4\pi K_2 H(U_1 - U_0)} < 1 \qquad (33)$$

and K_2/K_1 is about 1,000, σ is very small, and U_{ext} is close to U_0 for r only slightly $> R(t)$.

Consequently, the solution is closely given by

$$U = U_0 + \frac{S}{4\pi K_2 H} e^{-r^2/4K_1 t} \quad \text{for } r \leqq R(t) \tag{34}$$

$$= U_0 \quad \text{for } r > R(t) \tag{35}$$

Moreover, the derivative of U with respect to r is small for all r and t except for r or t close to 0. Thus, U can be taken as substantially constant for $r \leqq R(t)$ except close to the leak or during early stages of formation of the plume. This result is essentially the same as given by the Isaacson model.

For small leak rates, the confined plume departs from the Isaacson model since no leak front develops. However, the significant quantity is

$$V = F(\beta)\, U \tag{36}$$

where

$$\beta = \int r\, U\, dr \tag{37}$$

This is due to the fact that detection of the leak depends on V rather than U.

Now $F(\beta)$ at time $t = 0$ is 0 for $r > 0$, and is 1 for $r = 0$. Integrating for β gives

$$\beta = \frac{U_0}{2} r^2 - \frac{St}{2\pi H} e^{-r^2/4K_1 t} \tag{38}$$

so that

$$\left.\begin{array}{l} \beta = 0 \quad \text{when } r = 0,\ t = 0 \\ \beta > 0 \quad \text{when } r > 0,\ t = 0 \end{array}\right\} \tag{39}$$

Consequently,

$$\left.\begin{aligned} F(\beta) &= 1 \quad \text{when } \beta \leq 0 \\ F(\beta) &= 0 \quad \text{when } \beta > 0. \end{aligned}\right\} \tag{40}$$

That is,

$$V = 0 \text{ for } \frac{U_0}{2} r^2 > \frac{St}{4\pi H} e^{-r^2/4K_1 t} \tag{41}$$

and

$$V = U_{int} \text{ for } \frac{U_0}{2} r^2 \leq \frac{St}{4\pi H} e^{-r^2/4K_1 t} \tag{42}$$

This means that V shows a leak front; the previous argument then applies to V to show that the Isaacson model is a good approximation.

Solution for the Unconfined Model

The interior solution for n = 3 is

$$U_{int} = U_0 + \frac{S}{2\pi^{3/2} K_2} \cdot \frac{1}{r} \int_{r/\sqrt{4K_2 t}}^{\infty} e^{-s^2} ds \tag{43}$$

Using the notation

$$\text{erfc}\,(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-s^2} ds \tag{44}$$

the solution is

$$U_{int} = U_0 + \frac{S}{4\pi K_2} \cdot \frac{1}{r} \,\text{erfc} \left(\frac{r}{\sqrt{4K_2 t}}\right) \tag{45}$$

The function erfc (x) has the following behavior:

$$\text{erfc}\ (0) = 1 \tag{46}$$

$$\text{erfc}\ (x) \approx \frac{e^{-x^2}}{x^2} \text{ as } x \to \infty \tag{47}$$

Consequently, for $U_1 > U_0$ (which is always the case) the equation

$$U_1 = U_0 + \frac{S}{4\pi K_2} \cdot \frac{1}{r} \cdot \text{erfc}\ \frac{(r)}{\sqrt{4K_2 t}} \tag{48}$$

or

$$\frac{1}{r}\ \text{erfc}\ \frac{(r)}{\sqrt{4K_2 t}} = \frac{4\pi K_2(U_1 - U_0)}{S} \tag{49}$$

always has a unique solution $r = R(t)$. Moreover, as $t \to \infty$ so that $\frac{r}{\sqrt{4K_2 t}} \to 0$, the equation becomes $\frac{1}{r} = \frac{4\pi K_2(U_1 - U_0)}{S}$. Thus, $R(t)$ tends to $R_0 = \frac{S}{4\pi K_2(U_1 - U_0)}$ as a limit.

A natural length R_0 is thus defined and a natural time t_0 is given by

$$\frac{{R_0}^2}{4K_2 t_0} = 1 \text{ or } t_0 = \frac{{R_0}^2}{4K_2}\ .$$

A lengthy computation which will not be given here shows that:

(1) For $t < t_0$, almost all the leak fluid is confined behind the leak front at $t = t_0$.

(2) Initially $R(t)$ expands at the rate const. $\sqrt{\left(\frac{t}{\log\ (t_0/t)}\right)}$, which is very much faster than $\sqrt{t}$ (or $\sqrt[3]{t}$)

(3) For $t > t_0$, $R(t)$ is approximately $R(t) \cong R_0 \cdot (1-\sqrt{t_0/t})$.

(4) For $t > 3t_0$, $R(t)$ can be taken as approximately R_0; the fluid leaks through the leak front at the same rate at which it enters the leak.

An approximate value for R_0 and t_0 can be obtained with the typical values

$S = 0.03$ gal/min $= 1.9$ cm^3/sec

$U_1 = 0.10$

$U_0 = 0.05$

$K_2 = 10^{-2}$ cm^2/sec.

Then $R_0 = 30$ cm $\cong 1$ ft and $t_0 = 22{,}500$ sec $\cong$ 6-1/2 hr. If $U_1 = 0.10$ and $U_0 = 0.09$ then $R_0 = 5$ ft and $t_0 = 153$ hr $\cong$ 1 week. Thus, for dry wells outside a distance of 5 ft from the leak, only the exterior solution is significant.

The exterior solution for $t > t_0$ can be taken in the form (for some σ)

$$U_{ext} = U_0 + \frac{\sigma}{4K_1\pi} \cdot \frac{1}{r} \operatorname{erfc}\left(\frac{r}{\sqrt{4K_1 t}}\right)$$

where the rate of leakage at $r = R_0$ is that produced by the motion of leak fluid through the sphere at a constant rate $S/4\pi R_0^2$, for $t > t_0$. That is

$$U_1 = U_0 + \frac{\sigma}{4\pi K_1} \cdot \frac{1}{R_0} \operatorname{erfc}\left(\frac{R_0}{\sqrt{4K_1 t}}\right) \tag{50}$$

and since $\operatorname{erfc}(R_0/\sqrt{4K_1 t}) \cong 1$ for $t > t_0$,

$$\sigma = 4\pi K_2(U_1 - U_0)\, R_0 \tag{51}$$

$$R_0 = \frac{S}{4\pi K_2(U_1 - U_0)} \tag{52}$$

$$\sigma = \frac{K_1}{K_2} S \qquad (53)$$

Thus

$$U_{ext} = U_0 + \frac{S}{4\pi K_2} \cdot \frac{1}{r} \operatorname{erfc}\left(\frac{r}{\sqrt{4K_1 t}}\right) \qquad (54)$$

Consequently, the total solution for U is

(1) For $t \leqq t_0$,

$$U = U_0 + \frac{S}{4\pi K_2} \cdot \frac{1}{r} \operatorname{erfc}\left(\frac{r}{\sqrt{4K_2 t}}\right) \text{ for } r \leqq R(t) \qquad (55)$$

$$U = U_0 \text{ for } r > R(t) \qquad (56)$$

(2) For $t > t_0$,

$$U = U_0 + \frac{S}{4\pi K_2} \cdot \frac{1}{r} \operatorname{erfc}\left(\frac{r}{\sqrt{4K_2 t}}\right) \text{ for } r \leqq R_0 \qquad (57)$$

$$U = U_0 + \frac{S}{4\pi K_2} \cdot \frac{1}{r} \operatorname{erfc}\left(\frac{r}{\sqrt{4K_1 t}}\right) \text{ for } r > R_0 \qquad (58)$$

The special solution β of the differential equation for A is then, for $r > R_0$,

$$\beta = \int r^2 U \, dr = \frac{U_0}{3} r^3 + \frac{S}{4\pi K_2} \int r \operatorname{erfc}\left(\frac{r}{\sqrt{4K_1 t}}\right) dr \qquad (59)$$

$$= \frac{U_0}{3} r^3 + \frac{S}{4\pi K_2} \cdot 4K_1 t \int \frac{r}{\sqrt{4K_1 t}} \operatorname{erfc}\left(\frac{r}{\sqrt{4K_1 t}}\right) d \frac{r}{\sqrt{4K_1 t}} \qquad (60)$$

$$= \frac{U_0}{3} r^3 + \frac{K_1 S}{K_2} t \int^{s\, =\, r/\sqrt{4K_1 t}} s \text{ erfc } (s) \text{ ds} \qquad (61)$$

Now

$$\int s \text{ erfc } (s) \text{ ds} = \frac{2}{\sqrt{\pi}} \; {}^{s}\left(\int_s^\infty e^{-x^2} \text{ dx}\right) \text{ ds} \qquad (62)$$

and an integration by parts yields

$$\frac{2}{\sqrt{\pi}} \int s \left(\int_s^\infty e^{-x^2} \text{ dx}\right) \text{ ds} = \left(\frac{s^2}{2} - \frac{1}{4}\right) \text{erfc}(s) - \frac{s\, e^{-s^2}}{2\sqrt{\pi}} \qquad (63)$$

so

$$\beta = \frac{U_0}{3} r^3 + \frac{K_1 S}{K_2} t \cdot \left[\left(\frac{r^2}{8K_1 t} - \frac{1}{4}\right) \text{erfc } \left(\frac{r}{\sqrt{4U_1 t}}\right) - \frac{r e^{-r^2/4U_1 t}}{2\sqrt{4\pi U_1 t}}\right] \qquad (64)$$

$$= \frac{U_0}{3} r^3 + \left(\frac{r^2 S}{8K_2} - \frac{K_1 S}{4K_2 t}\right) \text{erfc } \left(\frac{r}{\sqrt{4K_1 t}}\right) - \frac{U_1}{\sqrt{\pi K_2}} \frac{r}{4U_2\sqrt{t}} \cdot e^{-r^2/U_1 X_1 t} \qquad (65)$$

Then $A = F(\beta)$. Now for $t = 0$, $A = A_0$ at $r = 0$ and $A = 0$ for $r > 0$. Also

$$\beta = \frac{U_0}{3} r^3 \text{ for } t = 0 \qquad (66)$$

Consequently, $F(\beta) = A_0$ for $\beta \leqq 0$ and $F(\beta) = 0$ for $\beta > 0$.

Now for t large, both

$$\text{erfc } \left(\frac{r}{\sqrt{4K_1 t}}\right) \qquad (67)$$

and

$$e^{-r^2/4K_1t} \qquad (68)$$

are close to 1, while $r/\sqrt{4K_1t}$ is close to 0. Thus

$$\beta \approx \frac{U_0}{3} r^3 - \frac{K_2S}{2K_1} t \qquad (69)$$

and hence $\beta = 0$ for

$$r = \frac{\sqrt[3]{3K_2S}}{2K_1U_0} \cdot \sqrt[3]{t} \qquad (70)$$

Again $\partial U/\partial r$ is small, so the result for V is that

$$V = C \text{ for } r \leqq C_1\sqrt[3]{t} \qquad (71)$$

$$V = 0 \text{ for } r \geqq C_1\sqrt[3]{t} \qquad (72)$$

where C and C_1 are appropriate constants. When t is large and r is larger than a few feet. This is again essentially the same result as obtained by the Isaacson model.

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